Investigating the Volume and Mass of Tyrannosaurus rex Specimen Sue (FMNH PR2081)

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INTRODUCTION

For as long as I can remember, I've had interest in animals, and especially dinosaurs. As I've grown, I've started to get more interested in the science behind dinosaur reconstructions. Upon seeing a new paper on different ways to estimate dinosaur mass (Dempsey, et. al), the method of Mathematical Double Integration to calculate volume and thus mass stood out to me, as it was closely linked to mathematics. This method is relatively easy, and provides estimates with a maximum error of around 20%, far more accurate than allometric methods which rely on large data sets and can have huge error margins (Wedel).

I wanted to apply this method of mass calculation to the most popular dinosaur: Tyrannosaurus rex. The availability of specimens and data for comparison made the Tyrannosaurus a good opportunity to experiment with double integration as a tool for calculating dinosaur mass.

AIM OF EXPLORATION

The aim of this exploration is to calculate the volume of a Tyrannosaurus rex using mathematical methods to create an estimate for mass, and comparing the results with contemporary estimates.

BACKGROUND INFORMATION

Tyrannosaurus rex was one of, if not the largest terrestrial carnivore. It was a tyrannosaurid theropod that lived during the Late Cretaceous period. It is known from over two dozen specimens, including several juvenile specimens (Britannica).

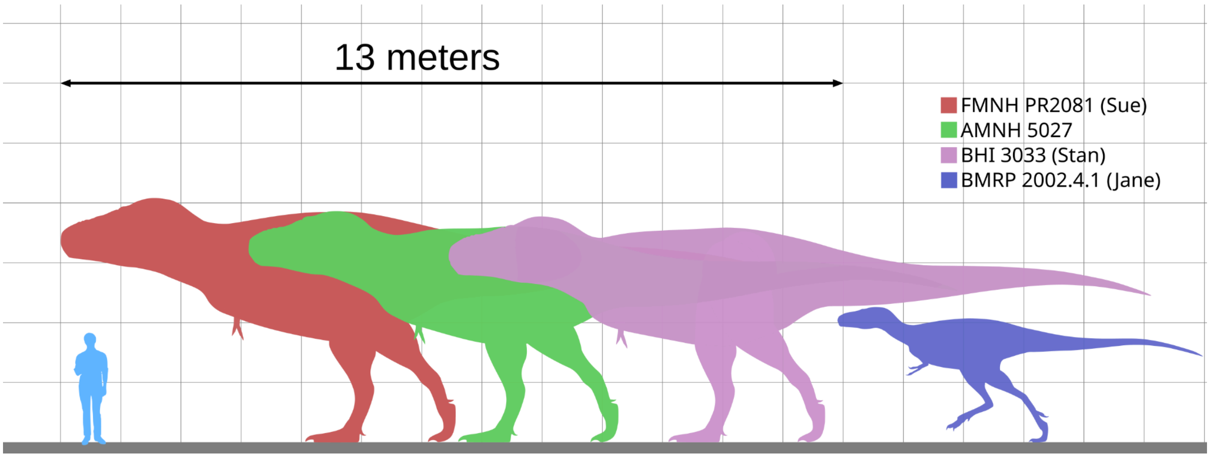

Fig. 1 Several large T. rex specimens, as well as a juvenile specimen, scaled with an average person (Wikimedia Commons).

Using fossils, it is possible to give a mass estimate for an animal, which is essential for understanding the biology of an animal. However, these techniques are not perfect, and some assumptions have to be made about soft tissues, such as fat and muscle, which are not preserved in fossils (Dempsey, 2). Most studies use formulae derived from large data sets that use the femur circumference to give a mass estimate. While useful for generalizations, these formulae can have a very large margin of error and cannot inform about the mass distribution across the animal.

Using a model of the skeleton, methods such as making a digital volumetric could lead to more accurate and specific results. Likewise, splitting the model into segments and using graphical double integration can be used to create a mass estimate. While being more of a simplification than digital volumetric modelling, it is a relatively quick way to estimate the mass of any one individual and allows adjusting mass distribution to some extent.

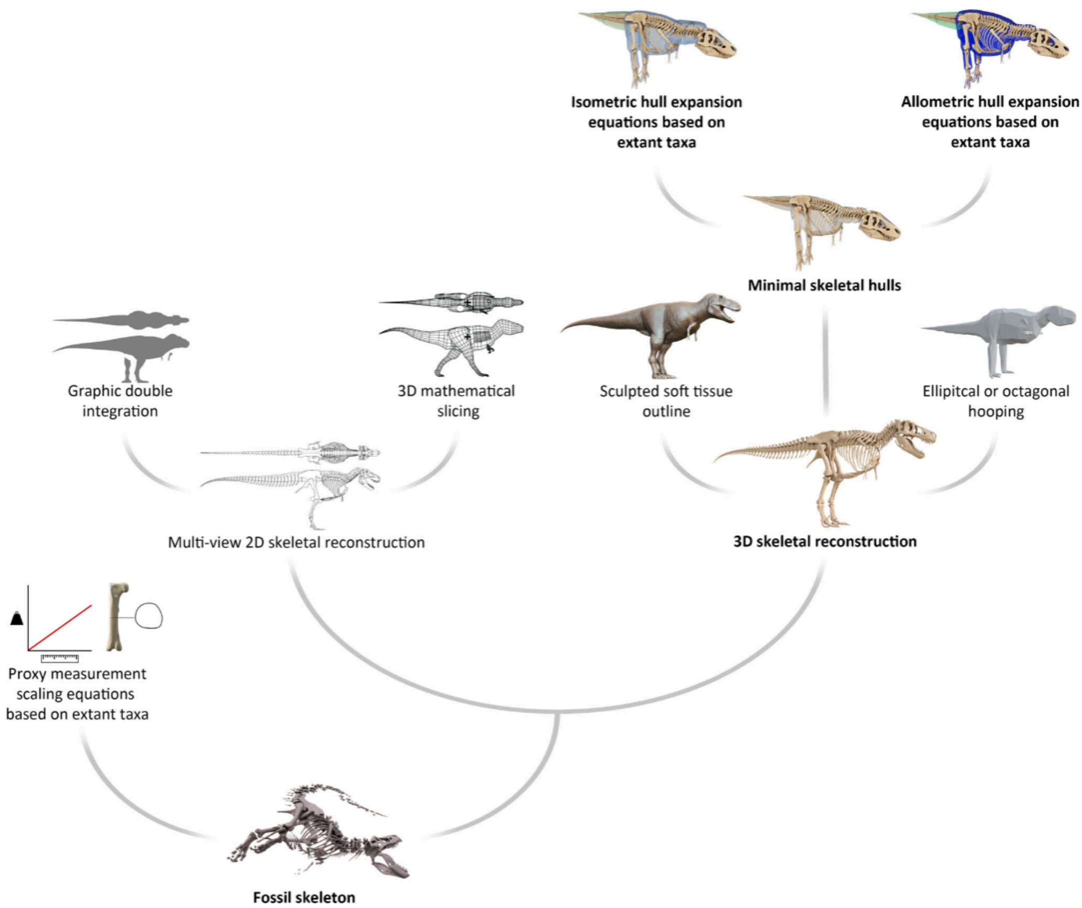

Fig. 2 The different methods of estimating the mass of a fossil animal (Dempsey, 1831)

For my calculations, I will use a specific individual as the basis for my measurements. I will use the specimen FMNH PR2081, nicknamed Sue, which is one of the largest and most complete Tyrannosaurus fossils known (Field Museum). She is estimated to be around 12.4 meters in length and 4 meters in height (Field Museum) and her weight has been estimated at around 9.5 metric tonnes using volumetric 3D models (Hutchinson, 9). Using several mathematical methods

METHODOLOGY

Mathematical Double Integration, or MDI, used in paleontological studies, differs somewhat from actual double integration but it follows a similar philosophy. The side view of the animal is mapped out and split into equally sized bars. These bars added together roughly equate the area of the side profile of the animal, with the bars of smaller width giving a more precise area.

This is the same method used in integration, where bars of height f(x) and width of dx, which have an infinitesimal distance of x, are summed together to find the exact area under a function.

It can be assumed that the animal has elliptical cross-sections, which while not completely accurate, is close enough for mass estimates. Using top and side views of the animal, the radius of these elliptical cross-sections can be determined. By multiplying the width of the bar with the ellipse for each section, the animal can be split into many cylinders that are roughly similar to its shape. The sum of the volumes of these cylinders can be used to determine the total volume of the animal, which can be multiplied by a density to determine the mass (Wedel).

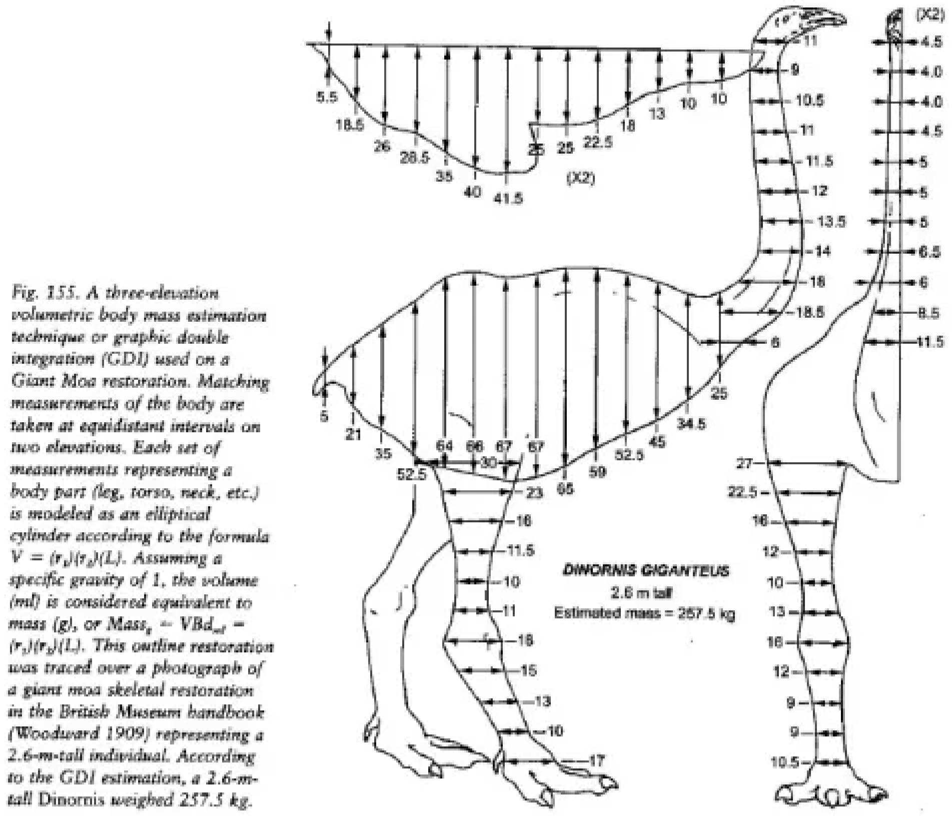

Fig. 10 An example of Mathematical Double Integration used to find the volume of Dinornis, a giant species of elephant bird (Wedel).

However, this method can be quite time consuming as it involves the summation of a lot of parts. Especially so when calculating the volumes of multiple individuals. To use as an alternative to MDI, I decided to use the proper formula for double integration to find a 3D surface roughly in the shape of a torso, and then the volume between it and the x-y plane. I will be referring to this as “True Double Integration”, or TDI.

MDI splits the animal into its main parts: the torso, arms and legs; and then further splits those parts into smaller cylindrical sections. TDI instead aims to find the volume of a singular part with one calculation. This means that the total volume can be found with far fewer calculations. This method directly relies on the formula for double integration, and how it is used to calculate volumes.

The formula for the volume of a shape with a surface defined by the surface function h(x,y) bound by the functions f(x) and g(x) can be written as a double integral:

$$Volume = \int_{b}^{a} \int_{g(x)}^{f(x)} h(x,y)dydx$$

Where a and b are two constant x values. Additionally, a>b and f(x) > g(x) and both f(x) and g(x) are continuous along $a \leq x \leq b$. Initially the integral for y is calculated, assuming x is a constant. This creates one area under the surface, like a single integral. Then this area is multiplied by the infinitely small width dx and infinite amount of times, creating a volume that sweeps under the surface.

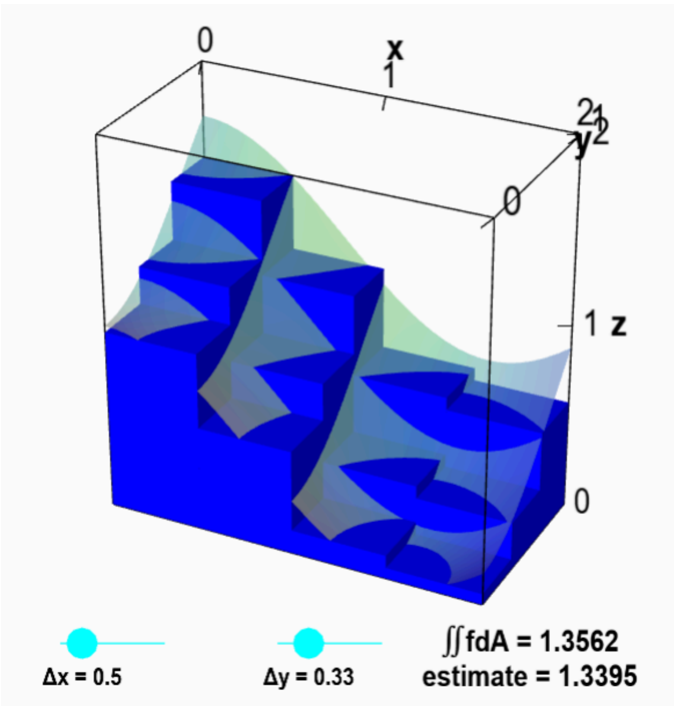

Fig. 11 An example of how double integration approximates volume (Math Insight)

Figure 11 shows how double integration works. Summing together the volume of the blue boxes is quite close to the volume under the smooth surface. If these boxes are infinitely small in width and depth and infinitely many, then their summed volume will be exactly that of the volume under the surface.

To test the possibilities of TDI in calculating dinosaur mass, I want to estimate the mass of the trunk of Sue, which comprises the head, torso and tail and should make up the majority of her mass (Wedel). To do this I will make a diagram of Sue viewed from the side and from the top. By putting points along the outline of this silhouette I can fit a polynomial function on the points using the software Geogebra.

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The side view of Sue will be split into two functions, f(x) and g(x), which will be the determining boundaries for the surface, h(x,y). The surface will be modelled using the function for the top-view of Sue, and assuming that her body is shaped somewhat parabolically.

The final volume will then be multiplied by $0.95g/cm^3 = 950kg/m^3$, which is the estimated internal density for large theropods (Reolid, 6). I will also account for the mass of the limbs which wasn't modelled and the percentage of empty internal spaces in the torso, such as the lungs, using approximations.

MODELLING SUE

Using the estimated sizes, side and top profiles were constructed using the vector art program Inkscape. The profiles were constructed on top of a grid to determine the correct proportions.

Using a side profile to create functions defined as y = f(x) and using a top-view profile to create functions defined as x = g(y), it is possible to create a surface function z = h(x,y). Double integration will be used to find the volume of these surface functions. By summing together the volumes of all the individual parts, it will be possible to determine the volume for one individual.

Figure 10 shows the block-out model for Sue, based on a skeletal diagram by Scott Hartman. Although the figure has legs and arms modelled too, their volume will not be calculated for the sake of simplicity. Although the figure is very simplified compared to the original skeletal diagram, this shouldn't impact measurements much as the surface function will also be an approximation, so only the rough proportion should matter. The block-out model was made to account for the curved tail and open mouth on the original diagram, which would have complicated the surface function used in calculations.

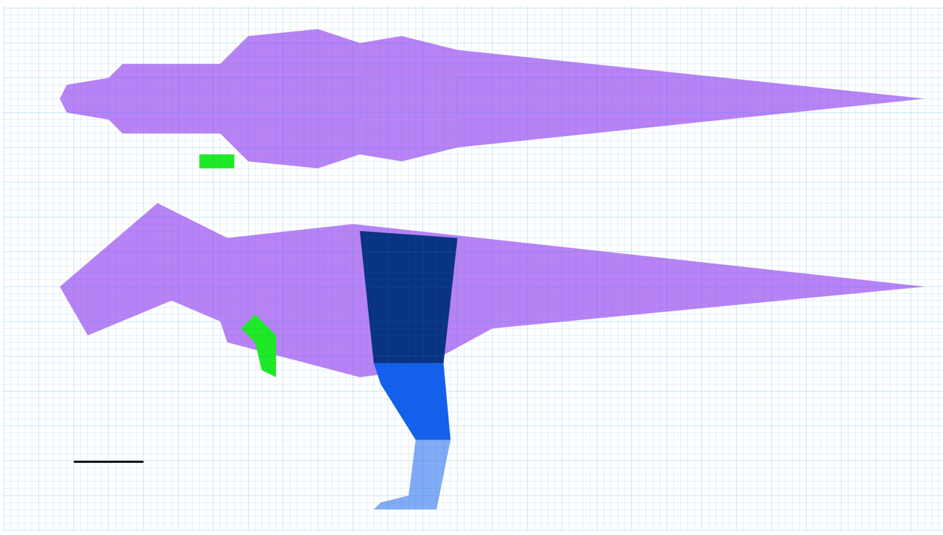

Fig. 12 A block-out of the side and top view of Sue. The scale bar is 1 meter and the smallest squares on the grid are 1 decimeter.

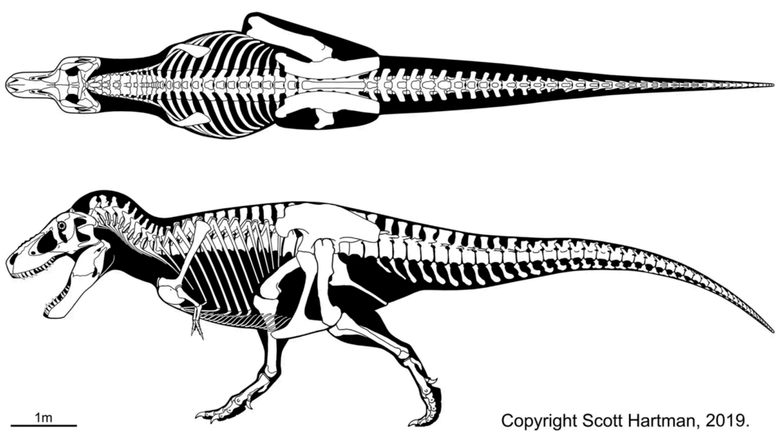

Fig. 13 Skeletal diagram of Sue (Hartman)

MODELLING FUNCTIONS

The trunk was imported into Geogebra, making sure to keep as close as possible to her estimated length, where 1 unit represented 1 meter. Based on this I added many points to create an outline for her upper and lower halves. Using the Polyfit command, I could model functions for each half that were relatively close to the outline. Her upper half is defined as f(x) and her lower half as g(x).

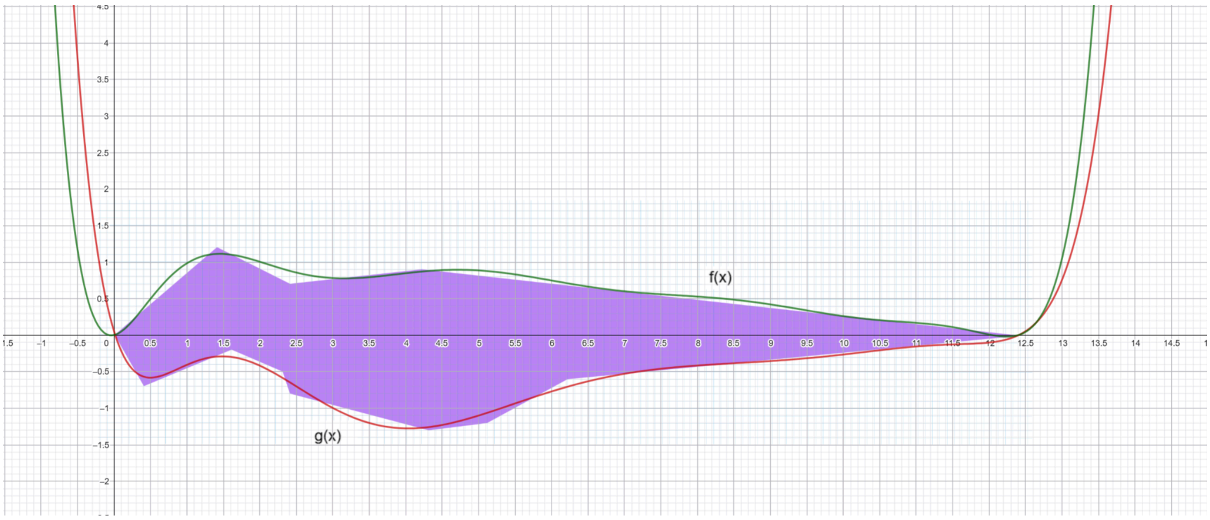

Fig 14. The functions f(x) and g(x) modelled in Geogebra.

The functions were only modelled along the interval $0 \leq x \leq 12.4$. The functions were both polynomials to the tenth degree as this gave the line closest to the silhouette, They were kept at 15 decimal points as rounding the functions any lower caused significant distortions.

$$f(x) = 3.24800403 * 10^{-7}x^{10} - 2.1189951136 * 10^{-5}x^{9} + 5.90666525567 * 10^{-4}x^{8}$$

$$- 0.00916461324153x^{7} + 0.086324504335279x^{6} - 0.504048349729336x^{5}$$

$$+ 1.778020898447905x^{4} - 3.467134398849088x^{3} + 2.866257622698913x^{2}$$

$$+ 0.231545210156644x + 0.003955650057121$$

$$g(x) = 1.28502136 * 10^{-7}x^{10} - 9.004888029 * 10^{-6}x^{9} + 2.75946613242 * 10^{-4}x^{8}$$
$$- 0.004832657154034x^{7} + 0.052918315084523x^{6} - 0.371359787466452x^{5}$$
$$+ 1.639696024987679x^{4} - 4.271173111919627x^{3} + 5.759630914054864x^{2}$$
$$- 3.262668470136394x + 0.053654659377978$$

Initially, I struggled on how I could define a surface h(x,y) required to complete the double integral. A surface function has both an x and y value for any point, and for each coordinate point on the 2-dimensional grid there is a height value for the third axis, z.

I noticed that when a surface function is defined as z = a(x) + b(y) where a(x) is a surface function where x is mapped to z and b(y) is a surface function where y is mapped to z, then the resulting surface is a combination of both functions.

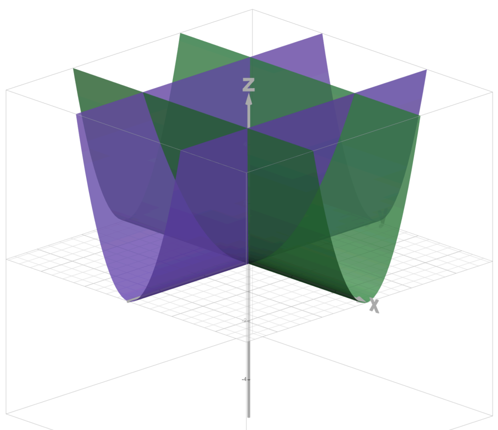

Fig. 15 The functions $z = x^2$ *and* $z = y^2$

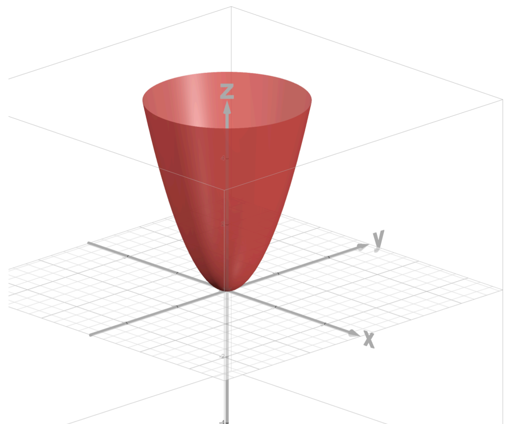

Fig. 16 The function $z = x^2 + y^2$

Figure 15 shows two parabolic surfaces, one extending infinitely in the x direction and the other extending infinitely in the y direction. However, in figure 16 the individual functions are added together to create a single function, which now looks like a rounded cone. Using a similar method, if I were to sum c(x), which would be the function of the top view of Sue, and d(y), a negative parabolic function with roots approximately as far apart as Sue would be wide, I could create a surface function:

$$h(x, y) = c(x) + d(y)$$

First I modelled a function for the top view of Sue. As tetrapods tend to have bilateral symmetry, I only needed to model half of the top view, as the other half would be the same. Like the functions f(x) and g(x) for the side view, I also modelled these in Geogebra and fit a polynomial function along the points.

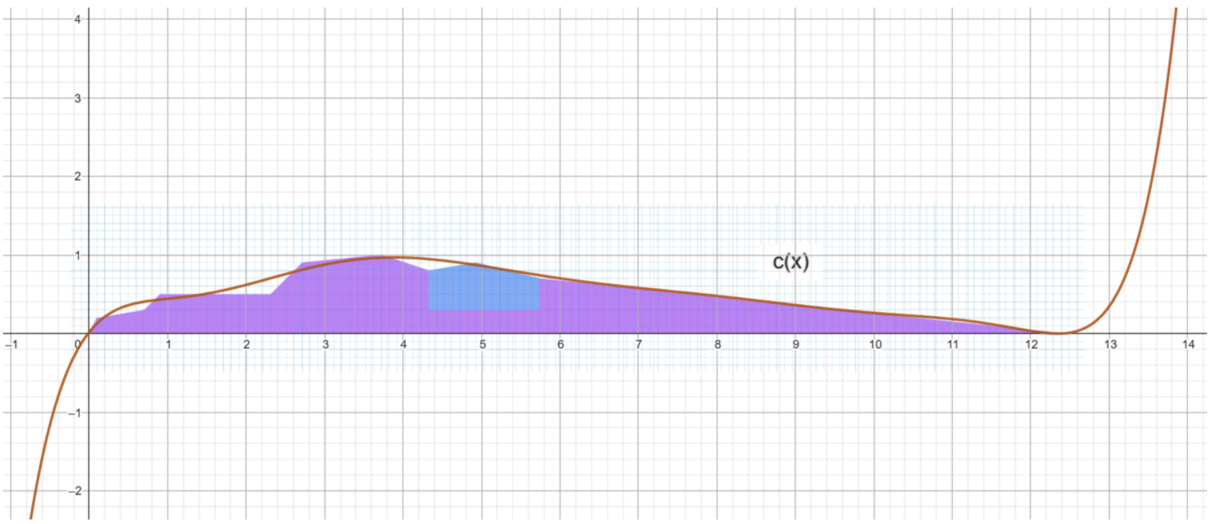

Fig. 17 The function c(x) modelled as a y function

For simplicity, the function was modelled as a function that maps y to x, instead of z to y. However, for final calculations the function was adjusted to be a function of z.

$$c(x) = 1.8132993 * 10^{-8}x^{10} - 6.10784059 * 10^{-7} * x^{9} - 1.23033933 * 10^{-6}x^{8}$$

$$3.49031171588 * 10^{-4}x^{7} - 0.006992049481117x^{6} + 0.06745109533577x^{5}$$

$$- 0.359972984391277x^{4} + 1.055053461747075x^{3} - 1.584787240358965x^{2}$$

$$+ 1.251310655551923x + 0.021132800274183$$

The function d(y) was modelled in a much simpler fashion. I initially modelled it as a function of y mapped to x. However for the final formula I replaced y with z and x with y. The final formula is a parabolic surface defined as:

$$z = -0.7y^2$$

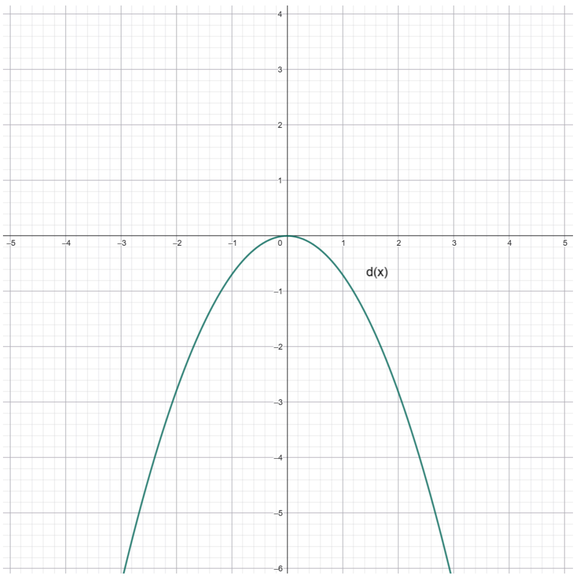

Fig. 18 The function d(x) before being converted to a surface function

As h(x,y) = c(x) + d(y), the final function is written as:

$$h(x, y) = 1.8132993 * 10^{-8}x^{10} - 6.10784059 * 10^{-7}x^{9} - 1.23033933 * 10^{-6}x^{8}$$

$$3.49031171588 * 10^{-4}x^{7} - 0.006992049481117x^{6} + 0.06745109533577x^{5}$$

$$- 0.359972984391277x^{4} + 1.055053461747075x^{3} - 1.584787240358965x^{2}$$

$$+ 1.251310655551923x + 0.021132800274183 - 0.7y^{2}$$

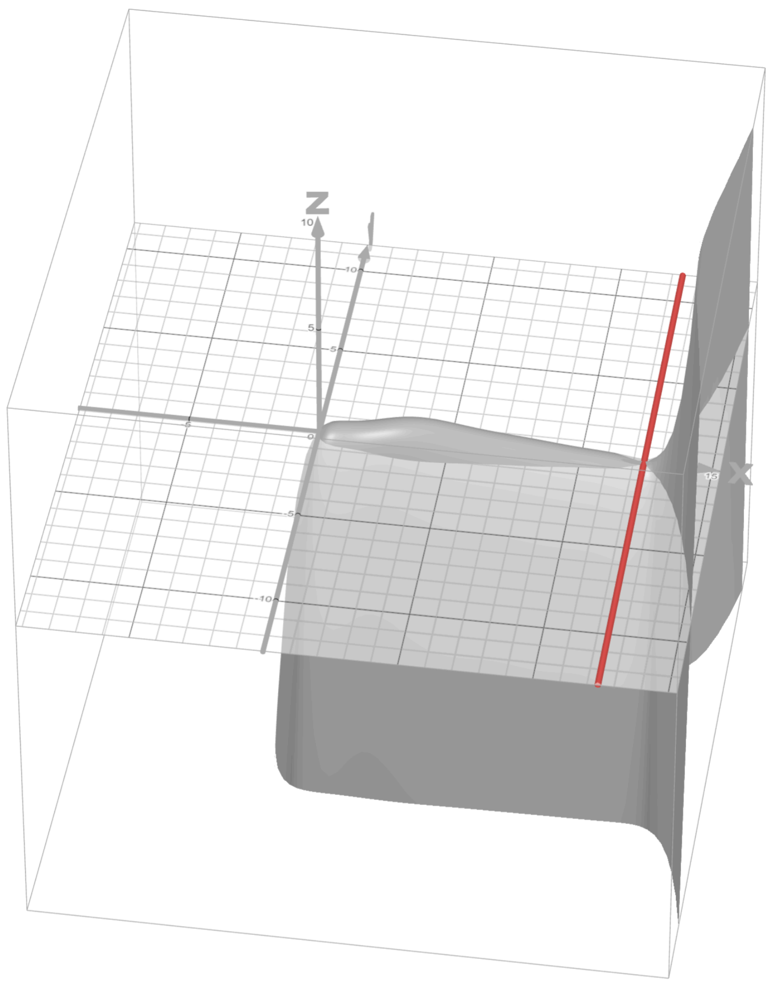

Fig. 19 The surface function h(x,y)

Figure 19 shows the appearance of the surface function h(x,y). The red line is x=12.4 and it is used to show the limit of the function as it is only considered on the interval $0 \leq x \leq 12.4$ and $z \geq 0$.

FINAL CALCULATIONS

Now that we have the 2 functions to define the boundaries of the double integral, and the surface function which we will find the area under, it is possible to calculate a double integral, whose volume should be equivalent to half the volume of the trunk of Sue.

However, as all the polynomials are to the tenth degree, the function is very complicated. To simplify calculations, I split the integral into two parts. The first part calculates the volume of the upper half and the second part the volume of the lower half. As the lower half is below the x-axis, the absolute value of the integral is taken due to the volume being negative.

$$Volume,\ V = \int_{0}^{12.4} \int_{g(x)}^{f(x)} h(x,y)dydx = \int_{0}^{12.4} \int_{0}^{f(x)} h(x,y)dydx + \left| \int_{0}^{12.4} \int_{0}^{g(x)} h(x,y)dydx \right|$$

Even when simplified, the equation is far too complicated to calculate by hand, as each variable in the surface function $h(x, y)$ would be substituted with a polynomial $f(x)$ and $g(x)$ itself. While technically possible, this would lead to massive amounts of data and would risk errors if done by hand. Instead, I used the open-source program *wxMaxima* to calculate the integral. However, I encountered some issues when calculating between the interval $0 \leq x \leq 12.4$ as the program would have too many figures to compute as the program would give a “floating point overflow” error. Due to this reason, I reduced the interval to $0 \leq x \leq 12$, which solved the issue. However, this would slightly reduce the calculated mass, as it means that a portion of the tail is missing from the final calculation. However, as the bulk of the volume is in the torso, the overall impact should be minimal. The final formula I used to calculate the volume in wxMaxima was:

$$Volume,\ V = \int_{0}^{12} \int_{0}^{f(x)} h(x,y)dydx + \left| \int_{0}^{12} \int_{0}^{g(x)} h(x,y)dydx \right|$$

The results of the integrals are in meters cubed. The final volume of the trunk is equivalent to the sum of the two integrals multiplied by 2.

$$V = 2 * (3.648134341371097m^3 + \left|- 3.5775007888256063m^3\right|) = 2 * 7.225635130196704m^3$$

$$14.451270260393407m^3 \approx 14.5m^3$$

For consistency, all final results will be rounded to 3 significant figures. Unrounded values will be used for calculations.

Using the density of $\rho = 950kgm^{-3}$, we can calculate the mass of the torso. This should give the mass.

$$Torso\ mass,\ m = V\rho = 14.451270260393407m^3 * 950kgm^{-3} = 13728.706747373737kg$$

$$m \approx 13.7 * 10^3 kg$$

Matt Wedel found that the torso, tail, neck and head made up 85% of the total mass when calculating the mass of Plateosaurus using MDI. As Tyrannosaurus has a far larger head and stockier neck than Plateosaurus, I assumed that the tail, neck and head made up about 95% of the total mass. John Hutchinson estimates that around 5.6% or the mass of Sue consisted of the legs excluding the thighs and the forelimbs (Hutchinson). The thighs are included in the volume of the trunk. Using this assumption, I can get a rough estimate for the total mass of Sue.

$$Total\ mass,\ m_t = \frac{m}{0.9} = 14451.270260393409kg \approx 14.4 * 10^3 kg$$

However, this calculation doesn't account for any air pockets inside the body, such as the lungs and windpipe. Generally dinosaurs and their relatives are very pneumatized with numerous air-sacs within their bodies. This would have reduced their mass, and is one of the adaptations that allowed dinosaurs to reach such large sizes.

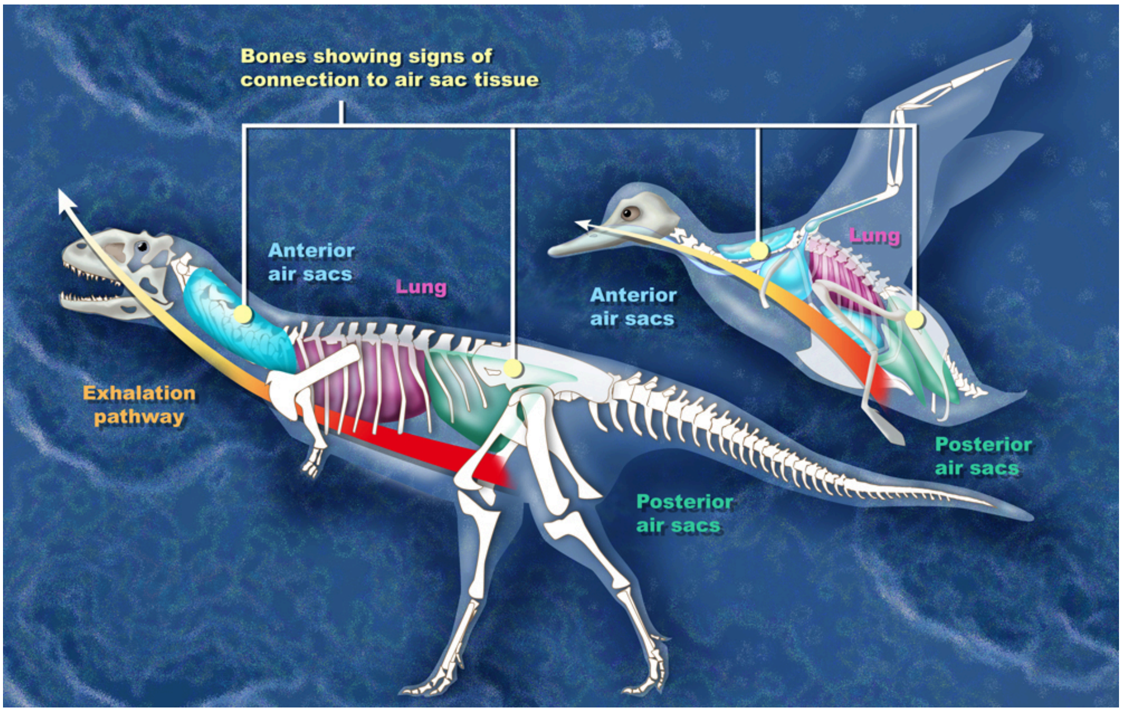

Fig. 20 A diagram showing the air sac systems within two dinosaurs: the extinct Majungasaurus and a bird (Deretsky)

These air sacs can be assumed to be "zero density spaces" with no mass (Hutchinson, 2). These would lower the average density, and for Sue, Hutchinson estimates the average density would be $791kgm^{-3}$ (Hutchinson, 4). Using this value instead of the initial one results in a mass estimate of:

$$Mass\ of\ torso\ using\ density\ of\ 791kgm^{-3},\ m_2\ =\ V\rho_2 = 14.451270260393407m^3 * 791kgm^{-3}$$

$$m_2 = 11430.95477597118kg \approx 11.4 * 10^3 kg$$

$$Total\ mass\ using\ density\ of\ \ 791kgm^{-3},\ m_{t2} = \frac{m_2}{0.9} = 12032.583974706513kg \approx 12.0 * 10^3 kg$$

All these calculated values can be converted into metric tonnes by dividing the value by 1000.

$$Mass\ of\ torso\ \ using\ density\ of\ 950kgm^{-3},\ m \approx 13.7\ t$$

$$Total\ mass\ using\ density\ of\ 950kgm^{-3},\ m_t \approx 14.4\ t$$

$Mass\ of\ torso\ using\ density\ of\ 791kgm^{-3},\ m_2 \approx\ 11.4\ t$

$Total\ mass\ using\ density\ of\ \ 791kgm^{-3},\ m_{t2} \approx\ 12.0\ t$

EVALUATION

There were a lot of assumptions used when calculating the volume and mass of Sue. The proportions in general were somewhat simplified, and especially when calculating volume these differences might have had a very large effect on the final result (Wedel). A simple model can show that even a 10% difference in the proportions can lead to a 30% effect on the calculated volume. Where the original proportion is 1 meter, if the incorrect one is 1.1 meters the calculated volume would be:

$(1.1m)^3 = 1.33m^3$ Which is about 30% greater than $1m^3$, which is the actual volume.

In addition to this, there were a decent amount of assumptions about the density and amount of soft tissues in Sue, which is an inherent issue in any mass estimate based solely on fossils.

While the initial value of 14.4 tonnes is a very large mass estimate, it still falls within general mass estimates for Tyrannosaurus which range from 5.66 tonnes to 10.9 tonnes (Dempsey, 10), with an average around 9 tonnes for an adult individual. Hutchinson's estimates for Sue using 3D volumetric models range from 9.5 tonnes all the way to 18.5 tonnes.

Despite being a higher estimate, the secondary mass estimate of 12.0 tonnes, which takes into account air sacs, is a relatively reasonable approximation for the mass of Sue, despite being on the greater end of the range of estimates.

CONCLUSION

Overall, this was a very unique experience and a way for me to learn about the science behind dinosaurs, especially when it comes to applying mathematics into the field. I believe that the TDI mass estimation system has potential thanks to simplifying the body into certain parts and could even be used in proper scientific studies, albeit with more rigor than my process, such as calculating the masses of the limbs as well.

When compared to other methods, such as MDI, TDI has some advantages but also some downsides. It has the potential to be a much quicker way to calculate mass, especially if the approximation of functions based on body diagrams can be automated in some way. However, I also found that it was very difficult to find a function that fit as closely as possible, which is why the values I used were kept at a very high 10 decimal points where values with only 2 or 3 decimal points would have been more appropriate considering the precision of the other values used. A possible alternative would be to use piecewise functions instead of a single function. This could yield good results while being quite simple and could still be quite accurate if exponential functions are used in the piecewise function, although this would require more time spent on calculations due to having to separate the model into many parts. Lastly, the most simple method would be to approximate the model into simple boxes and wedges and summing their volumes. Unless the boxes are very small however, this simplification might lead to inaccurate results.

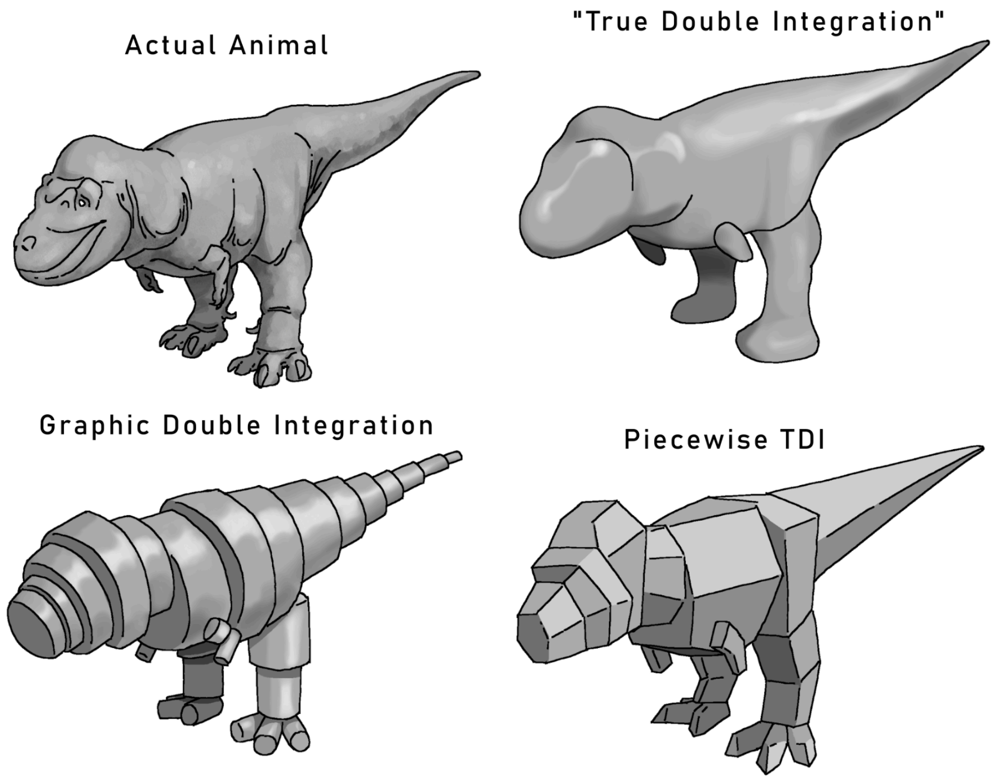

Fig. 21 A simplified visualization of the different methods of modelling a Tyrannosaurus.

Figure 21 highlights some of the advantages and disadvantages of each method. While TDI is quick and mostly effective, it can be difficult to accurately model small details, such as toes. GDI and TDi using piecewise functions give more unnatural blocky results, but can still give mostly accurate mass estimates.Albeit these two processes are more tedious due to many repetitive calculations being involved. Overall, TDI is a relatively easy yet accurate mass estimation method but is limited by the difficulty of mapping a function onto the shape of an animal.

In conclusion, using mathematical models to explain real-world phenomena has always been a subject I enjoy. This investigation was especially rewarding as it allowed me to develop my own method of mass calculation for dinosaurs.

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